Solution to HW #2

3-4

Step 1: Find R_A & R_E

$$h = \frac{4.5}{\tan 30^\circ} = 7.794 \text{ m}$$

$\Sigma M_A = 0$

$9R_E - 7.794(400\cos 30^\circ)$

$-4.5(400\sin 30^\circ) = 0$

$R_E = 400 \text{ N}$ *Ans.*

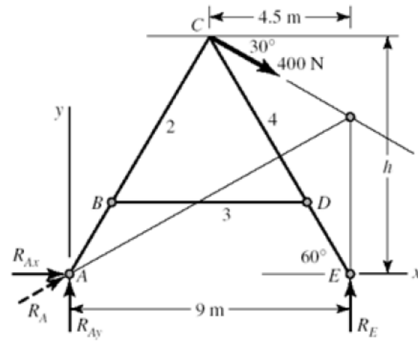
$\sum F_x = 0 \quad R_{Ax} + 400\cos 30^\circ = 0$

$R_{Ax} = -346.4 \text{ N}$

$\sum F_y = 0 \quad R_{Ay} + 400 - 400\sin 30^\circ = 0$

$R_{Ay} = -200 \text{ N}$

$R_A = \sqrt{346.4^2 + 200^2} = 400 \text{ N}$ *Ans.*



Step 2: Find components of R_C on link 4 and R_D

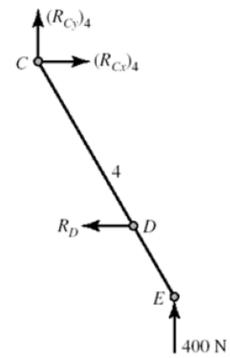
$\sum M_C = 0$

$400(4.5) - (7.794 - 1.9)R_D = 0$

$R_D = 305.4 \text{ N}$ *Ans.*

$\sum F_x = 0 \quad \Rightarrow \quad (R_{Cx})_4 = 305.4 \text{ N}$

$\sum F_y = 0 \quad \Rightarrow \quad (R_{Cy})_4 = -400 \text{ N}$



Step 3: Find components of R_C on link 2

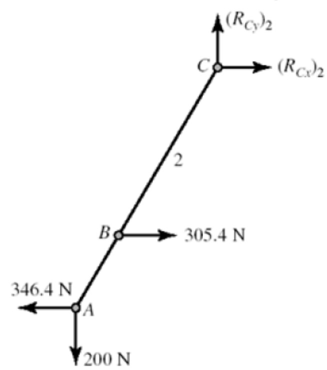
$\sum F_x = 0$

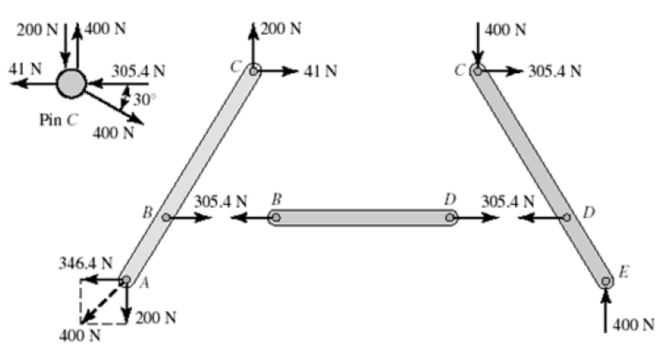
$(R_{Cx})_2 + 305.4 - 346.4 = 0$

$(R_{Cx})_2 = 41 \text{ N}$

$\sum F_y = 0$

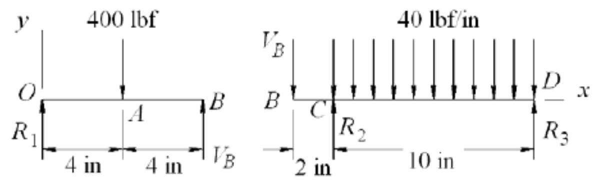
$(R_{Cy})_2 = 200 \text{ N}$





3-8

Break at the hinge at B



Beam OB:

From symmetry,

$R_1 = V_B = 200$ lbf   *Ans.*

Beam BD:

$\Sigma M_D = 0$

$200(12) - R_2(10) + 40(10)(5) = 0$

$R_2 = 440$ lbf   *Ans.*

$\Sigma F_y = 0$

$-200 + 440 - 40(10) + R_3 = 0$

$R_3 = 160$ lbf   *Ans.*

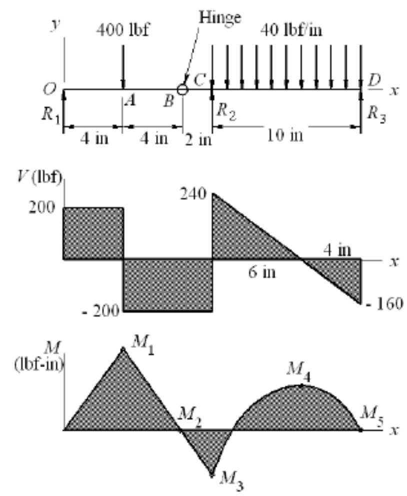
$M_1 = 200(4) = 800$ lbf · in   *Ans.*

$M_2 = 800 - 200(4) = 0$ checks at hinge

$M_3 = 800 - 200(6) = -400$ lbf · in   *Ans.*

$$M_4 = -400 + \frac{1}{2}(240)(6) = 320 \text{ lbf} \cdot \text{in} \quad \textit{Ans.}$$

$$M_5 = 320 - \frac{1}{2}(160)(4) = 0 \quad \text{checks!}$$



3-16

(a)

$$C = \frac{-8+7}{2} = -0.5 \text{ MPa}$$

$$CD = \frac{8+7}{2} = 7.5 \text{ MPa}$$

$R = \sqrt{7.5^2 + 6^2} = 9.60$ MPa

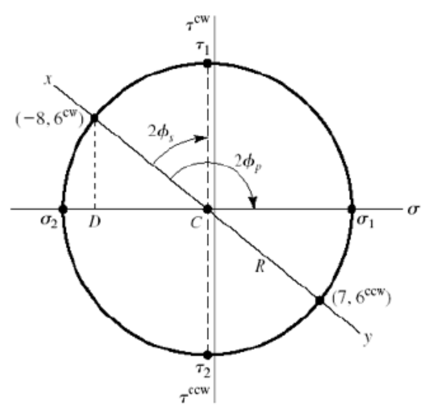
$\sigma_1 = 9.60 - 0.5 = 9.10$ MPa

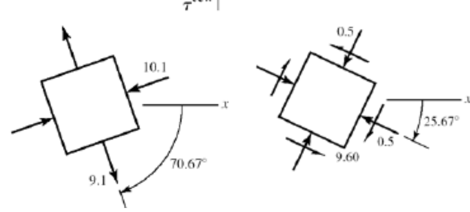
$\sigma_2 = -0.5 - 9.6 = -10.1$ Mpa

$$\phi_p = \frac{1}{2}\left[90^\circ + \tan^{-1}\left(\frac{7.5}{6}\right)\right] = 70.67^\circ \text{ cw}$$

$\tau_1 = R = 9.60$ MPa

$\phi_s = 70.67^\circ - 45^\circ = 25.67^\circ$ cw





(b)

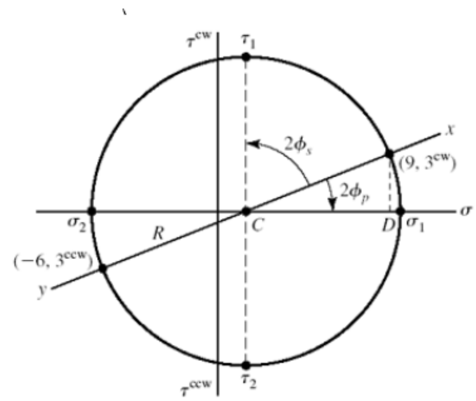
$$C = \frac{9-6}{2} = 1.5 \text{ MPa}$$

$$CD = \frac{9+6}{2} = 7.5 \text{ MPa}$$

$$R = \sqrt{7.5^2 + 3^2} = 8.078 \text{ MPa}$$

$$\sigma_1 = 1.5 + 8.078 = 9.58 \text{ MPa}$$

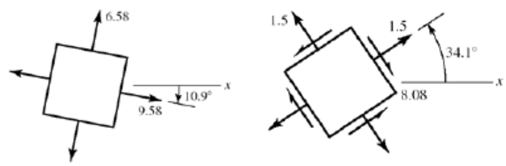
$$\sigma_2 = 1.5 - 8.078 = -6.58 \text{ MPa}$$



$$\phi_p = \frac{1}{2}\tan^{-1}\left(\frac{3}{7.5}\right) = 10.9^\circ \text{ cw}$$

$$\tau_1 = R = 8.078 \text{ MPa}$$

$$\phi_s = 45^\circ - 10.9^\circ = 34.1^\circ \text{ ccw}$$



(c)

$$C = \frac{12-4}{2} = 4 \text{ MPa}$$

$$CD = \frac{12+4}{2} = 8 \text{ MPa}$$

$$R = \sqrt{8^2 + 7^2} = 10.63 \text{ MPa}$$

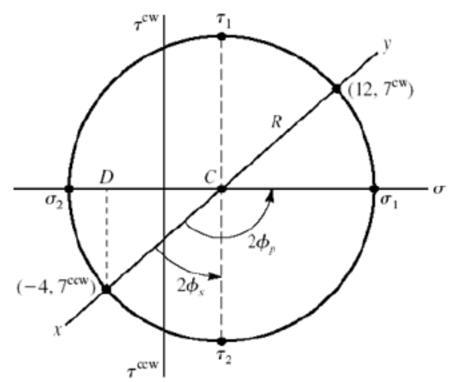
$$\sigma_1 = 4 + 10.63 = 14.63 \text{ MPa}$$

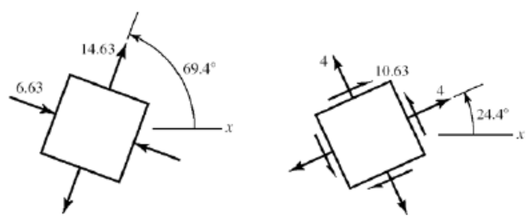
$$\sigma_2 = 4 - 10.63 = -6.63 \text{ MPa}$$

$$\phi_p = \frac{1}{2}\left[90^\circ + \tan^{-1}\left(\frac{8}{7}\right)\right] = 69.4^\circ \text{ ccw}$$

$$\tau_1 = R = 10.63 \text{ MPa}$$

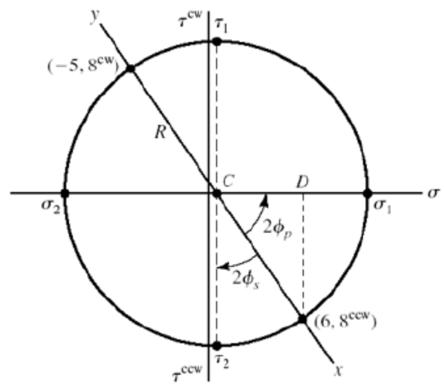
$$\phi_s = 69.4^\circ - 45^\circ = 24.4^\circ \text{ ccw}$$



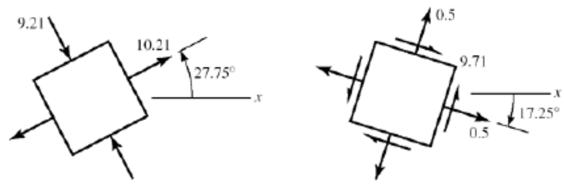


(d)

$$C = \frac{6-5}{2} = 0.5 \text{ MPa}$$
$$CD = \frac{6+5}{2} = 5.5 \text{ MPa}$$
$$R = \sqrt{5.5^2 + 8^2} = 9.71 \text{ MPa}$$
$$\sigma_1 = 0.5 + 9.71 = 10.21 \text{ MPa}$$
$$\sigma_2 = 0.5 - 9.71 = -9.21 \text{ MPa}$$



$$\phi_p = \frac{1}{2}\tan^{-1}\left(\frac{8}{5.5}\right) = 27.75^\circ \text{ ccw}$$
$$\tau_1 = R = 9.71 \text{ MPa}$$
$$\phi_s = 45^\circ - 27.75^\circ = 17.25^\circ \text{ cw}$$



3-25

$$\epsilon_2 = \frac{\Delta d}{d} = \frac{-0.0001d}{d} = -0.0001$$

From Table A-5, $\nu = 0.326$, $E = 119$ GPa

$$\epsilon_1 = \frac{-\epsilon_2}{\nu} = \frac{-0.0001}{0.326} = 306.7\left(10^{-6}\right)$$

$$\delta = \frac{FL}{AE} \text{ and } \sigma = \frac{F}{A}, \text{ so}$$

$$\sigma = \frac{\delta E}{L} = \epsilon_1 E = 306.7\left(10^{-6}\right)(119)\left(10^9\right) = 36.5 \text{ MPa}$$

$$F = \sigma A = 36.5\left(10^6\right)\frac{\pi(0.03)^2}{4} = 25\ 800 \text{ N} = 25.8 \text{ kN} \qquad \textit{Ans.}$$

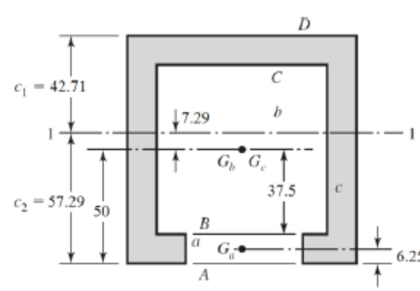
$S_y = 70$ MPa $> \sigma$, so elastic deformation assumption is valid.

3-34

(c)

Use two negative areas.

$$A_a = 625 \text{ mm}^2,\ A_b = 5625 \text{ mm}^2,\ A_c = 10\,000 \text{ mm}^2$$
$$A = 10\,000 - 5625 - 625 = 3750 \text{ mm}^2;$$



$\bar{y}_a = 6.25$ mm, $\bar{y}_b = 50$ mm, $\bar{y}_c = 50$ mm

$$\bar{y} = \frac{10\,000(50) - 5625(50) - 625(6.25)}{3750} = 57.29 \text{ mm} \quad \textit{Ans.}$$

$$c_1 = 100 - 57.29 = 42.71 \text{ mm} \quad \textit{Ans.}$$

$$I_a = \frac{50(12.5)^3}{12} = 8138 \text{ mm}^4$$

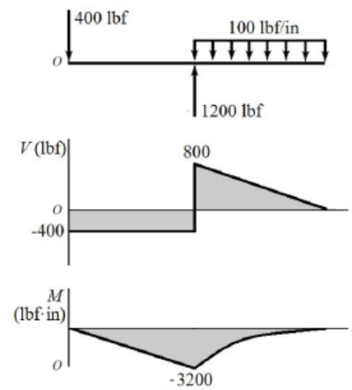
$$I_b = \frac{75(75)^3}{12} = 2.637\left(10^6\right) \text{ mm}^4$$

$$I_c = \frac{100(100)^3}{12} = 8.333\left(10^6\right) \text{in}^4$$

$$I_1 = \left[8.333\left(10^6\right) + 10\,000(7.29)^2\right] - \left[2.637\left(10^6\right) + 5625(7.29)^2\right] - \left[8138 + 625(57.29 - 6.25)^2\right]$$

$$I_1 = 4.29\left(10^6\right) \text{ in}^4 \quad \textit{Ans.}$$

3-36



$$I = \frac{1}{12}(1)(2)^3 = 0.6667 \text{ in}^4$$

$$A = 1(2) = 2 \text{ in}^2$$

$$\Sigma M_o = 0$$

$$8R_A - 100(8)(12) = 0$$

$$R_A = 1200 \text{ lbf}$$

$$R_o = 1200 - 100(8) = 400 \text{ lbf}$$

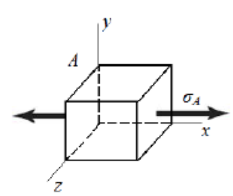
$M_{\max}$ is at A. At the top of the beam,

$$\sigma_{\max} = \frac{Mc}{I} = \frac{3200(0.5)}{0.6667} = 2400 \text{ psi} \quad \textit{Ans.}$$

Due to V, $\tau_{\max}$ is at A, at $y = 0$.

$$\tau_{\max} = \frac{3}{2}\frac{V}{A} = \frac{3}{2}\left(\frac{800}{2}\right) = 600 \text{ psi} \quad \textit{Ans.}$$

3-45 **(a)** $L = 10$ in. Element A:

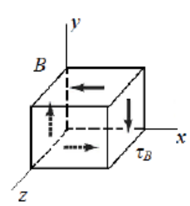


$$\sigma_A = -\frac{My}{I} = -\frac{-(1000)(10)(0.5)}{(\pi/64)(1)^4}\left(10^{-3}\right) = 101.9 \text{ kpsi}$$

$$\tau_A = \frac{VQ}{Ib}, \quad Q = 0 \;\Rightarrow\; \tau_A = 0$$

$$\tau_{\max} = \sqrt{\left(\frac{\sigma_A}{2}\right)^2 + \tau_A^2} = \sqrt{\left(\frac{101.9}{2}\right)^2 + (0)^2} = 50.9 \text{ kpsi} \quad \textit{Ans.}$$

Element B:



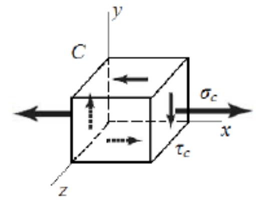
$$\sigma_B = -\frac{My}{I}, \quad y = 0 \;\Rightarrow\; \sigma_B = 0$$

$$Q = \bar{y}'A' = \left(\frac{4r}{3\pi}\right)\left(\frac{\pi r^2}{2}\right) = \frac{4r^3}{6} = \frac{4(0.5)^3}{6} = 1/12 \text{ in}^3$$

$$\tau_B = \frac{VQ}{Ib} = \frac{(1000)(1/12)}{(\pi/64)(1)^4(1)}\left(10^{-3}\right) = 1.698 \text{ kpsi}$$

$$\tau_{\max} = \sqrt{\left(\frac{0}{2}\right)^2 + 1.698^2} = 1.698 \text{ kpsi} \qquad \textit{Ans.}$$

Element C:

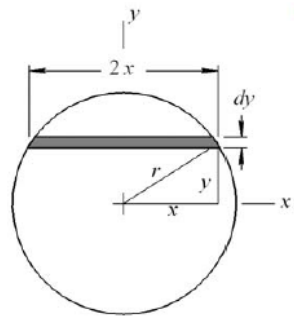


$$\sigma_C = -\frac{My}{I} = -\frac{-(1000)(10)(0.25)}{(\pi/64)(1)^4}\left(10^{-3}\right) = 50.93 \text{ kpsi}$$

$$Q = \int_{y_1}^{r} y\,dA = \int_{y_1}^{r} y(2x)\,dy = \int_{y_1}^{r} y\left(2\sqrt{r^2 - y^2}\right)dy$$

$$= -\frac{2}{3}\left(r^2 - y^2\right)^{3/2}\Big|_{y_1}^{r} = -\frac{2}{3}\left[\left(r^2 - r^2\right)^{3/2} - \left(r^2 - y_1^2\right)^{3/2}\right]$$

$$= \frac{2}{3}\left(r^2 - y_1^2\right)^{3/2}$$



For C, $y_1 = r/2 = 0.25$ in

$$Q = \frac{2}{3}\left(0.5^2 - 0.25^2\right)^{3/2} = 0.05413 \text{ in}^3$$

$$b = 2x = 2\sqrt{r^2 - y_1^2} = 2\sqrt{0.5^2 - 0.25^2} = 0.866 \text{ in}$$

$$\tau_C = \frac{VQ}{Ib} = \frac{(1000)(0.05413)}{(\pi/64)(1)^4(0.866)}\left(10^{-3}\right) = 1.273 \text{ kpsi}$$

$$\tau_{\max} = \sqrt{\left(\frac{50.93}{2}\right)^2 + (1.273)^2} = 25.50 \text{ kpsi} \qquad \textit{Ans.}$$

(b) Neglecting transverse shear stress:

Element A: Since the transverse shear stress at point A is zero, there is no change.

$\tau_{\max} = 50.9$ kpsi *Ans.*

% error = 0% *Ans.*

Element B: Since the only stress at point B is transverse shear stress, neglecting the transverse shear stress ignores the entire stress.

$$\tau_{\max} = \sqrt{\left(\frac{0}{2}\right)^2} = 0 \text{ psi} \qquad \textit{Ans.}$$

$$\% \text{ error} = \left(\frac{1.698 - 0}{1.698}\right) * (100) = 100\% \qquad \textit{Ans.}$$

Element C:

$$\tau_{\max} = \sqrt{\left(\frac{50.93}{2}\right)^2} = 25.47 \text{ kpsi} \qquad \textit{Ans.}$$

$$\% \text{ error} = \left(\frac{25.50 - 25.47}{25.50}\right) * (100) = 0.12\% \qquad \textit{Ans.}$$

(c) Repeating the process with different beam lengths produces the results in the table.

	Bending stress, σ (kpsi)	Transverse shear stress, τ (kpsi)	Max shear stress, τ_{max} (kpsi)	Max shear stress, neglecting τ, τ_{max} (kpsi)	% error
***L* = 10 in**					
A	102	0	50.9	50.9	0
B	0	1.70	1.70	0	100
C	50.9	1.27	25.50	25.47	0.12
***L* = 4 in**					
A	40.7	0	20.4	20.4	0
B	0	1.70	1.70	0	100
C	20.4	1.27	10.26	10.19	0.77
***L* = 1 in**					
A	10.2	0	5.09	5.09	0
B	0	1.70	1.70	0	100
C	5.09	1.27	2.85	2.55	10.6
***L* = 0.1in**					
A	1.02	0	0.509	0.509	0
B	0	1.70	1.70	0	100
C	0.509	1.27	1.30	0.255	80.4

Discussion:
The transverse shear stress is only significant in determining the critical stress element as the length of the cantilever beam becomes smaller. As this length decreases, bending stress reduces greatly and transverse shear stress stays the same. This causes the critical element location to go from being at point *A*, on the surface, to point *B*, in the center. The maximum shear stress is on the outer surface at point *A* for all cases except $L = 0.1$ in, where it is at point *B* at the center. When the critical stress element is at point *A*, there is no error from neglecting transverse shear stress, since it is zero at that location. Neglecting the transverse shear stress has extreme significance at the stress element at the center at point *B*, but that location is probably only of practical significance for very short beam lengths.

3-48 **(a)**

x-z plane

$$\Sigma M_O = 0 = 1.5(0.5) + 2(1.5)\sin(30^\circ)(2.25) - R_{2z}(3)$$

$$R_{2z} = 1.375 \text{ kN} \quad Ans.$$

$$\Sigma F_z = 0 = R_{1z} - 1.5 - 2(1.5)\sin(30^\circ) + 1.375$$

$$R_{1z} = 1.625 \text{ kN} \quad Ans.$$

x-y plane

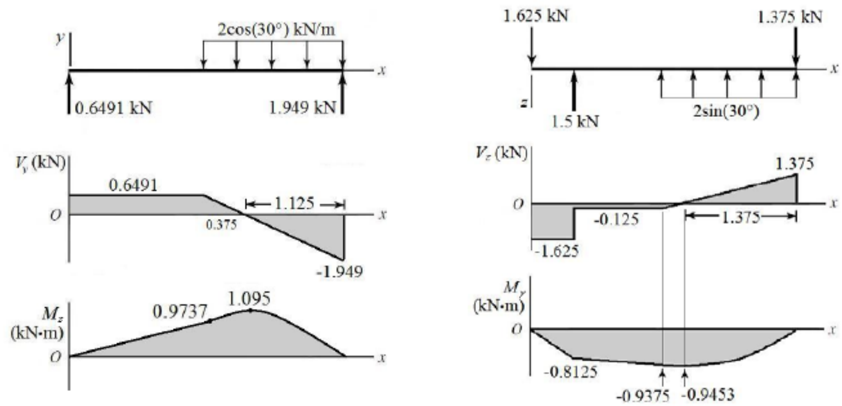
$$\Sigma M_O = 0 = -2(1.5)\cos(30^\circ)(2.25) + R_{2y}(3)$$

$$R_{2y} = 1.949 \text{ kN} \quad Ans.$$

$$\Sigma F_y = 0 = R_{1y} - 2(1.5)\cos(30^\circ) + 1.949$$

$$R_{1y} = 0.6491 \text{ kN} \quad Ans.$$

(b)



(c) The transverse shear and bending moments for most points of interest can readily be taken straight from the diagrams. For $1.5 < x < 3$, the bending moment equations are parabolic, and are obtained by integrating the linear expressions for shear. For convenience, use a coordinate shift of $x' = x - 1.5$. Then, for $0 < x' < 1.5$,

$$V_z = x' - 0.125$$

$$M_y = \int V_z dx' = \frac{(x')^2}{2} - 0.125x' + C$$

$$\text{At } x' = 0, M_y = C = -0.9375 \;\Rightarrow\; M_y = 0.5(x')^2 - 0.125x' + 0.9375$$

$$V_y = -\frac{1.949}{1.125}x' + 0.6491 = -1.732x' + 0.6491$$

$$M_z = \frac{-1.732}{2}(x')^2 + 0.6491x' + C$$

$$\text{At } x' = 0, M_z = C = 0.9737 \;\Rightarrow\; M_z = -0.8662(x')^2 - 0.125x' - 0.9375$$

By programming these bending moment equations, we can find M_y, M_z, and their vector combination at any point along the beam. The maximum combined bending moment is found to be at $x = 1.79$ m, where $M = 1.433$ kN·m. The table below shows values at key locations on the shear and bending moment diagrams.

x (m)	V_z (kN)	V_y (kN)	V (kN)	M_y (kN·m)	M_z (kN·m)	M (kN·m)
0	–1.625	0.6491	1.750	0	0	0
0.5^-	–1.625	0.6491	1.750	–0.8125	0.3246	0.8749
1.5	–0.1250	0.6491	0.6610	0.9375	0.9737	1.352
1.625	0	0.4327	0.4327	–0.9453	1.041	1.406
1.875	0.2500	0	0.2500	–0.9141	1.095	1.427
3^-	1.375	–1.949	2.385	0	0	0

(d) The bending stress is obtained from Eq. (3-27),

$$\sigma_x = \frac{-M_z y_A}{I_z} + \frac{M_y z_A}{I_y}$$

The maximum tensile bending stress will be at point A in the cross section of Prob. 3-34 (a), where distances from the neutral axes for both bending moments will be maximum. At A, for M_z, $y_A = -37.5$ mm, and for M_y, $z_A = -20$ mm.

$$I_z = \frac{40(75)^3}{12} - \frac{34(25)^3}{12} = 1.36(10^6)\ \text{mm}^4 = 1.36(10^{-6})\ \text{m}^4$$

$$I_y = 2\left[\frac{25(40)^3}{12}\right] + \frac{25(6)^3}{12} = 2.67(10^5)\ \text{mm}^4 = 2.67(10^{-7})\ \text{m}^4$$

It is apparent the maximum bending moment, and thus the maximum stress, will be in the parabolic section of the bending moment diagrams. Programming Eq. (3-27) with the bending moment equations previously derived, the maximum tensile bending stress is found at $x = 1.77$ m, where $M_y = -0.9408$ kN·m, $M_z = 1.075$ kN·m, and $\sigma_x = 100.1$ MPa. *Ans.*

3-57

(a) Obtain the torque from the given power and speed using Eq. (3-44).

$$T = 9.55\frac{H}{n} = 9.55\frac{(40\,000)}{2500} = 152.8\ \text{N}\cdot\text{m}$$

$$\tau_{\text{max}} = \frac{Tr}{J} = \frac{16T}{\pi d^3}$$

$$d = \left(\frac{16T}{\pi\tau_{\text{max}}}\right)^{1/3} = \left[\frac{16(152.8)}{\pi(70)(10^6)}\right]^{1/3} = 0.0223\ \text{m} = 22.3\ \text{mm} \quad Ans.$$

(b) $$T = 9.55\frac{H}{n} = 9.55\frac{(40\,000)}{250} = 1528\ \text{N}\cdot\text{m}$$

$$d = \left[\frac{16(1528)}{\pi(70)(10^6)}\right]^{1/3} = 0.0481\ \text{m} = 48.1\ \text{mm} \quad Ans.$$

3-71 **(a)**

$$T_2 = 0.15T_1$$

$$\sum T = 0 = (300-45)(125) + (T_2 - T_1)(150) = 31\,875 + (0.15T_1 - T_1)(150)$$

$$31\,875 - 127.5T_1 = 0 \qquad \Rightarrow \qquad T_1 = 250\ \text{N}\cdot\text{mm} \quad Ans.$$

$$T_2 = 0.15(250) = 37.5\ \text{N}\cdot\text{mm} \quad Ans.$$

(b)

$$\sum M_{Oy} = 0 = 345\sin 45^\circ(300) - 287.5(700) - R_{Cz}(850)$$

$$R_{Cz} = -150.7\ \text{N} \qquad Ans.$$

$$\sum F_z = 0 = R_{Oz} - 345\cos 45^\circ + 287.5 - 150.7$$

$$R_{Oz} = 107.2\ \text{N} \qquad Ans.$$

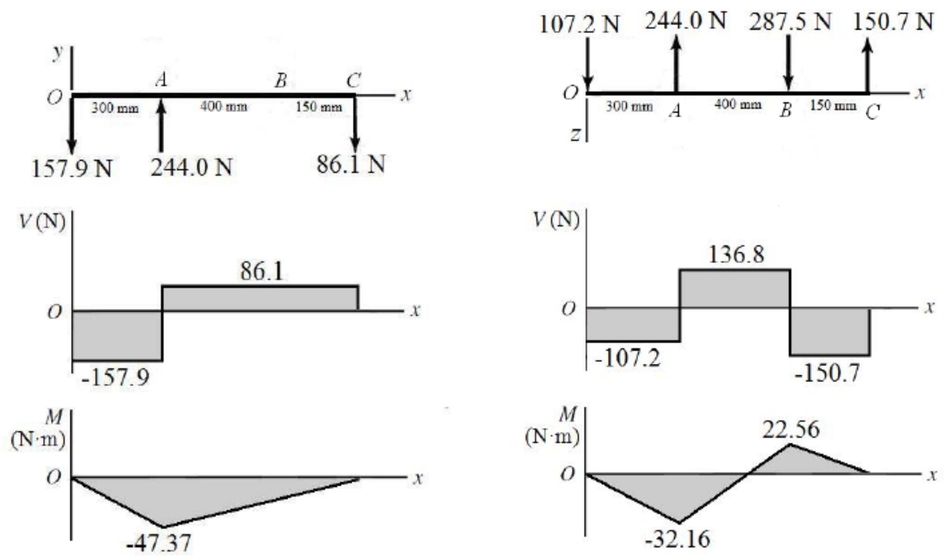
$$\sum M_{Oz} = 0 = 345\sin 45^\circ(300) + R_{Cy}(850)$$

$$R_{Cy} = -86.10\ \text{N} \qquad Ans.$$

$$\sum F_y = 0 = R_{Oy} + 345\cos 45^\circ - 86.10$$

$$R_{Oy} = -157.9\ \text{N} \qquad Ans.$$

(c)



n

ding moment diagrams, it is clear that the critical location is at A where both planes have the maximum bending moment. Combining the bending moments from the two planes,

$$M = \sqrt{(-47.37)^2 + (-32.16)^2} = 57.26 \text{ N}\cdot\text{m}$$

The torque transmitted through the shaft from A to B is $T = (300 - 45)(0.125) = 31.88$ N·m. For a stress element on the outer surface where the bending stress and the torsional stress are both maximum,

$$\sigma = \frac{Mc}{I} = \frac{32M}{\pi d^3} = \frac{32(57.26)}{\pi(0.020)^3} = 72.9(10^6) \text{ Pa} = 72.9 \text{ MPa} \qquad Ans.$$

$$\tau = \frac{Tr}{J} = \frac{16T}{\pi d^3} = \frac{16(31.88)}{\pi(0.020)^3} = 20.3(10^6) \text{ Pa} = 20.3 \text{ MPa} \qquad Ans.$$

(e)

$$\sigma_1,\ \sigma_2 = \frac{\sigma_x}{2} \pm \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + (\tau_{xy})^2} = \frac{72.9}{2} \pm \sqrt{\left(\frac{72.9}{2}\right)^2 + (20.3)^2}$$

$\sigma_1 = 78.2$ MPa *Ans.*

$\sigma_2 = -5.27$ MPa *Ans.*

$$\tau_{\max} = \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + (\tau_{xy})^2} = \sqrt{\left(\frac{72.9}{2}\right)^2 + (20.3)^2} = 41.7 \text{ MPa} \qquad Ans.$$

3-82

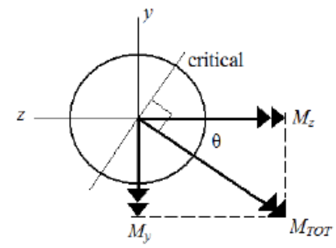
(a) Rod *AB* experiences constant torsion and constant axial tension throughout its length, and maximum bending moments at the wall from both planes of bending. Both torsional shear stress and bending stress will be maximum on the outer surface. The transverse shear will be very small compared to bending and torsion, due to the reasonably high length to diameter ratio, so it will not dominate the determination of the critical location. The critical stress element will be on the outer surface at the wall, with its critical location determined by the plane of the combined bending moments.

$M_y = -(100)(8) - (75)(5) = -1175$ lbf·in

$M_z = (-200)(8) = -1600$ lbf·in

$$M_{\text{tot}} = \sqrt{M_y^2 + M_z^2} = \sqrt{(-1175)^2 + (-1600)^2} = 1985 \text{ lbf}\cdot\text{in}$$

$$\theta = \tan^{-1}\left(\left|\frac{M_y}{M_z}\right|\right) = \tan^{-1}\left(\frac{1175}{1600}\right) = 36.3°$$



The combined bending moment vector is at an angle of 36.3° CW from the negative *z* axis. The critical bending stress location will be ±90° from this vector, as shown. Since there is an axial stress in tension, the critical stress element will be where the bending is also in tension. The critical stress element is therefore on the outer surface at the wall, at an angle of 36.3° CW from the *y* axis.

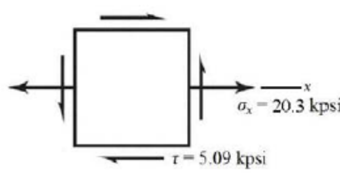
(b) Transverse shear is zero at the critical stress element on the outer surface.

$$\sigma_{x,\text{bend}} = \frac{M_{\text{tot}}c}{I} = \frac{M_{\text{tot}}(d/2)}{\pi d^4/64} = \frac{32M_{\text{tot}}}{\pi d^3} = \frac{32(1985)}{\pi(1)^3} = 20\,220 \text{ psi} = 20.2 \text{ kpsi}$$

$$\sigma_{x,\text{axial}} = \frac{F_x}{A} = \frac{F_x}{\pi d^2/4} = \frac{75}{\pi(1)^2/4} = 95.5 \text{ psi} = 0.1 \text{ kpsi}$$, which is essentially negligible

$$\sigma_x = \sigma_{x,\text{axial}} + \sigma_{x,\text{bend}} = 20\,220 + 95.5 = 20\,316 \text{ psi} = 20.3 \text{ kpsi}$$

$$\tau = \frac{Tr}{J} = \frac{16T}{\pi d^3} = \frac{16(5)(200)}{\pi(1)^3} = 5093 \text{ psi} = 5.09 \text{ kpsi}$$



(c)

$$\sigma_1,\ \sigma_2 = \frac{\sigma_x}{2} \pm \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau^2} = \frac{20.3}{2} \pm \sqrt{\left(\frac{20.3}{2}\right)^2 + (5.09)^2}$$

$\sigma_1 = 21.5$ kpsi *Ans.*

$\sigma_2 = -1.20$ kpsi *Ans.*

$$\tau_{\max} = \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau^2} = \sqrt{\left(\frac{20.3}{2}\right)^2 + (5.09)^2} = 11.4 \text{ kpsi}$$ *Ans.*

3-95 $\sigma_t > \sigma_l > \sigma_r$

$\tau_{\max} = (\sigma_t - \sigma_r)/2$ at $r = r_i$

$$\tau_{\max} = \frac{1}{2}\left[\frac{r_i^2 p_i}{r_o^2 - r_i^2}\left(1 + \frac{r_o^2}{r_i^2}\right) - \frac{r_i^2 p_i}{r_o^2 - r_i^2}\left(1 - \frac{r_o^2}{r_i^2}\right)\right] = \frac{r_i^2 p_i}{r_o^2 - r_i^2}\left(\frac{r_o^2}{r_i^2}\right) = \frac{r_o^2 p_i}{r_o^2 - r_i^2}$$

$$\Rightarrow \quad r_i = r_o\sqrt{\frac{(\tau_{\max} - p_i)}{\tau_{\max}}} = 100\sqrt{\frac{(25-4)10^6}{25\left(10^6\right)}} = 91.7 \text{ mm}$$

$t = r_o - r_i = 100 - 91.7 = 8.3$ mm *Ans.*

3-107 $\omega = 2\pi(2000)/60 = 209.4$ rad/s, $\rho = 3320\ 20$ kg/m^3, $\nu = 0.24$, $r_i = 0.01$ m, $r_o = 0.125$ m

Using Eq. (3-55)

$$\sigma_t = 3320(209.4)^2\left(\frac{3+0.24}{8}\right)\left[(0.01)^2 + (0.125)^2 + (0.125)^2 - \frac{1+3(0.24)}{3+0.24}(0.01)^2\right](10)^{-6}$$

$= 1.85$ MPa *Ans.*

3-126

(a) $\sigma = \pm\dfrac{Mc}{I} = \pm\dfrac{[3(4)][0.5(0.1094)]}{(0.75)\left(0.1094^3\right)/12} = \pm 8021 \text{ psi} = \pm 8.02 \text{ kpsi}$ *Ans.*

(b) $r_i = 0.125$ in, $r_o = r_i + h = 0.125 + 0.1094 = 0.2344$ in

From Table 3-4,

$$r_c = 0.125 + (0.5)(0.1094) = 0.1797 \text{ in}$$

$$r_n = \frac{0.1094}{\ln(0.2344/0.125)} = 0.174006 \text{ in}$$

$$e = r_c - r_n = 0.1797 - 0.174006 = 0.005694 \text{ in}$$

$$c_i = r_n - r_i = 0.174006 - 0.125 = 0.049006 \text{ in}$$

$$c_o = r_o - r_n = 0.2344 - 0.174006 = 0.060394 \text{ in}$$

$$A = bh = 0.75(0.1094) = 0.08205 \text{ in}^2$$

$$M = -3(4) = -12 \text{ lbf}\cdot\text{in}$$

The negative sign on the bending moment is due to the sign convention shown in Fig. 3-34. Using Eq. (3-65),

$$\sigma_i = \frac{Mc_i}{Aer_i} = \frac{-12(0.049006)}{0.08205(0.005694)(0.125)} = -10\ 070 \text{ psi} = -10.1 \text{ kpsi} \quad \textit{Ans.}$$

$$\sigma_o = -\frac{Mc_o}{Aer_o} = -\frac{-12(0.060394)}{0.08205(0.005694)(0.2344)} = 6618 \text{ psi} = 6.62 \text{ kpsi} \quad \textit{Ans.}$$

(c) $K_i = \dfrac{\sigma_i}{\sigma} = \dfrac{-10.1}{-8.02} = 1.26$ *Ans.*

$K_o = \dfrac{\sigma_o}{\sigma} = \dfrac{6.62}{8.02} = 0.825$ *Ans.*

3-134 From Eq. (3-68),

$$a = \sqrt[3]{\left(\frac{3F}{8}\right)\frac{\left(1-\nu_1^2\right)/E_1 + \left(1-\nu_2^2\right)/E_2}{1/d_1 + 1/d_2}}$$

$$a = \sqrt[3]{\left(\frac{3(10)}{8}\right)\frac{\left(1-0.292^2\right)/(207\,000) + \left(1-0.333^2\right)/(71\,700)}{1/25 + 1/40}} = 0.0990 \text{ mm}$$

From Eq. (3-69),

$$p_{\max} = \frac{3F}{2\pi a^2} = \frac{3(10)}{2\pi\left(0.0990^2\right)} = 487.2 \text{ MPa}$$

From Fig. 3-37, the maximum shear stress occurs at a depth of $z = 0.48\,a$.

$$z = 0.48a = 0.48(0.0990) = 0.0475 \text{ mm} \qquad \textit{Ans.}$$

The principal stresses are obtained from Eqs. (3-70) and (3-71) at a depth of $z/a = 0.48$.

$$\sigma_1 = \sigma_2 = -487.2\left\{\left[1 - 0.48\tan^{-1}(1/0.48)\right](1+0.333) - \frac{1}{2\left(1+0.48^2\right)}\right\} = -101.3 \text{ MPa}$$

$$\sigma_3 = \frac{-487.2}{1+0.48^2} = -396.0 \text{ MPa}$$

From Eq. (3-72),

$$\tau_{\max} = \frac{\sigma_1 - \sigma_3}{2} = \frac{(-101.3)-(-396.0)}{2} = 147.4 \text{ MPa} \qquad \textit{Ans.}$$

Note that if a closer examination of the applicability of the depth assumption from Fig. 3-37 is desired, implementing Eqs. (3-70), (3-71), and (3-72) on a spreadsheet will allow for calculating and plotting the stresses versus the depth for specific values of ν. For ν = 0.333 for aluminum, the maximum shear stress occurs at a depth of $z = 0.492a$ with $\tau_{\max} = 0.3025\, p_{\max}$.

This gives $\tau_{\max} = 0.3025\, p_{\max} = (0.3025)(487.2) = 147.38$ MPa. Even though the depth assumption was a little off, it did not have significant effect on the the maximum shear stress.